Evaluation of July 1, 2000 County and Municipal Population Estimates by the Arizona Department of Economic Security

Submitted to the
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I. Introduction

The State of Arizona uses annual estimates of population for its counties and municipalities in order to allocate state tax revenues and to set expenditure limits. According to a recent report by the Office of the Auditor General for the State of Arizona (2004) the state allocated $1.6 Billion in 2003 for partial revenue sharing to county and local governments on the basis of the population estimates. The important use of population estimates requires that they be of high quality. Specifically the population estimates need to be unbiased, with a small level of error, timely, and free from manipulation. The purpose of this evaluation project is to review the population estimates produced by the Arizona Department of Economic Security (DES) and POPTAC with respect to their quality. In this report, the methods and data used for estimating county and municipal population are described; the level of bias and accuracy in the official estimates for July 1, 2000 are presented; changes in the methods implemented since 2000 are evaluated, and comparisons are presented to estimates by the U.S. Census and previous estimates by DES.

The objective in developing a population estimate is to derive a value that would be produced if a full enumeration of the population were to be conducted. The population bases for estimates are data from the prior U.S. Census of Population and Housing. The bases are adjusted to reflect any changes to the official population made by the Census Bureau. Similarly the standard of comparison for evaluating population estimates is data from the latest Census of Population and Housing. The Census 2000 therefore serves as the standard of comparison for evaluating the AZDES estimates of state, county and place populations. The Census Bureau has not made any changes to the Census 2000 results for Arizona (US Census Bureau, 2004). A method for estimating population is based on a prior census and its accuracy evaluated against a subsequent census. The difference between an estimate and the census count for the same point in time is called the "error of closure."

II. Methods and Data

Since 1990, the Arizona Department of Economic Security (AZDES) has employed four methods for estimating population: Housing Unit; Ratio-Correlation; Vital Rates and Composite methods. The Housing Unit Method is the primary method. In estimating state and county population during the 1990s the Housing Unit Method was used in combination with the Ratio-Correlation Method. Following the estimates for 2000 and subsequent release of the Census 2000

counts, the DES revised their approach, replacing the Ratio-Correlation Method with the Composite Method to be used in combination with the Housing Unit Method. State and county estimates for 2001 and 2002 were based 100 percent on the Housing Unit Method. In the third year after the Census 2000 the Composite Method was added and given a weight of 20 percent versus 80 percent for the Housing Unit Method. In the fourth year the maximum weight for the Composite Method, 30 percent is reached, reducing the Housing Unit Method to 70 percent.

For sub-county areas--cities and towns--the Housing Unit Method has been used alone. A recent survey of state demographers (FSCPE, 2003) reported that the most common methods for estimating county population were the housing unit and component methods, followed by the ratio-correlation and composite methods. For sub-county estimates of municipalities the clear favorite was the housing unit method. The AZDES' choice of methods is in line with the methods employed by most states.

The Housing Unit Method is based on an accounting identity for the household population. The household population can be expressed as the product of the number of housing units that are occupied by the average household size. The total population is the sum of the household population and persons living in group quarters (e.g. college dormitories, prisons, nursing homes). The formula for the household population is:

$$HHP_t = HU_t \times OCC_t \times PPH_t$$

Where:

HHP_t = Population in households at time t
HU_t = Housing units at time t
OCC_t = Occupancy rate at time t
PPH_t = Persons per household (average household size) at time t

The AZDES implements the Housing Unit Method for post census years by estimating the number of housing units in a county or place for the estimates year, and holding constant the occupancy rate and persons per household as reported for the area in the prior census enumeration. The number of housing units is estimated by taking the certificates of occupancy plus mobile home placements, and subtracting the number of housing units for which demolition permits were issued. At the place level, annexations are used as well.

The number of persons living in group quarters are estimated separately, by a annual survey of counties and jurisdictions regarding the group quarters facilities located there, and added to the household population in order to arrive at an estimate of the total population.

The major weakness in the Housing Unit Method as implemented by the AZDES is the reliance on occupancy rates and average household size (persons per household) from the prior census. There has not been a cost effective way to update these factors for counties and municipalities. The U.S. Census Bureau uses the Housing Unit Method for their estimates of sub-county populations and relies on occupancy rates and persons per household from the prior census as well. In order to compensate for this weakness in the Housing Unit Method, the AZDES uses a second method as well and then combines the estimates. This procedure can be used for preparing the county estimates but is not available for the sub-county estimates.

County Estimates

In the decade of the 1990s, the Housing Unit Method was the core method for preparing estimates, supplemented with estimates from the Ratio-Correlation Method in a manner that gave most of the weight to the Housing Unit Method. The ratio-correlation multivariate regression model estimates the relationship of the variables to total population size. The ratio-correlation method used school enrollment, federal tax returns and driver license counts for the counties. In the years following the census, the updated values of these indicators are substituted into the estimated model and produce an estimate of total population. Following 2000 estimates and release of the Census 2000 counts, the estimates were based solely on the Housing Unit Method until the second year after the census, when estimates prepared with the Composite Method were slated to be added in. That year the Housing Unit Method was to be weighted 90 percent and the Composite Method 10 percent. However, the composite method had not been adequately developed and therefore the Housing Unit was again solely used. The weights shifted to 80 and 20 percent in the third year, and then to 70 and 30 percent for the fourth year as the Composite method was now used. The Composite Method also uses symptomatic indicators and establishes simple ratios between the values of an indicator and an age segment of the total population. The Composite Method uses the indicators of: births minus age less than 5 deaths related to the population under age 5; school enrollment related to the population aged 5 to 17; all driver licenses related to the population aged 18 to 64; and Medicare enrollment related to the population aged 65 and older.

The estimates produced by the Housing Unit Method and supplemental method, Ratio-Correlation or Composite, are combined in a weighted average. The weighting scheme for the estimates adjusts from full reliance on the Housing Unit Method estimates in the year immediately following the decennial census, to a weighted average by the fourth year following the census of 70 percent the Housing Unit Method estimates and 30 percent the Composite Method estimates.

Place Estimates

The estimates for sub-county areas are prepared using the Housing Unit Method alone. The inputs on building permits, mobile home placements, annexations and demolitions required by the Housing Unit Method are available for municipalities. The symptomatic indicators used in the Composite Method are not available for municipalities. The Housing Unit Method place estimates are proportionally adjusted to agree with the county population estimate.

III. Results

County Population Estimates for 2000

The standard used for determining error in the estimates was the Census 2000. A comparison of the county population estimates for July 1, 2000 and the census counts for April 1, 2000 are presented in Table 1. A small amount of error is due to the difference in reference dates between the estimates (July 1, 2000) and the Census 2000 (April 1, 2000). On a numeric basis the largest errors of estimate were for the larger counties, with Maricopa County being underestimated by -80,899 persons, which is greatest numeric difference. Pima County was overestimated by 22,379 persons, and was the largest positive numeric error. In terms of percentage differences the largest errors of estimate were generally for the smaller counties, with the notable exception of Yuma County. Yuma County had the largest percentage difference, -10.8 percent, and the third greatest numeric difference, -17,276. On a percentage basis the errors for Maricopa County, at -2.63 percent, and for Pima County, at 2.65 percent were relatively small. The estimate for Gila County differed from the census count by only -160 persons, or -0.3 percent and was the county with the smallest amount of error.

Place Population Estimates for 2000

Places ranged in population size, according to the Census 2000, between Phoenix at 1,321,045 and Jerome at 329. It was Jerome that among all places in the state had the greatest percentage error of estimate, a 76.3 percent overestimate. The numeric difference of 251 persons was small in absolute terms but great in relative terms. Whereas the error of estimate for Phoenix was far smaller on a percentage basis, -5.4 percent, it represented a substantial numeric difference of -71,595 persons. The error of estimate for thirteen places was less than 1 percent. The largest of these places was Tucson with a population of 486,699 and an error of estimate of 909 persons, for a -0.2 percent underestimate. A comparison of census counts and estimates for places is presented in Table 2.

Measures of Error

The estimate of population for the state was 85,357 below the census count. The errors of estimate for the counties, which were controlled to the state estimate, summed to the same difference. For an evaluation of the accuracy of county estimates, such a summing up does not express the total level of error as overestimates and underestimates cancel each other out. Instead error is more accurately described as the sum of the absolute differences. The sum of absolute differences between estimates and the census count for counties is 174,381. The sum of absolute differences for places was 169,070.

Errors for population estimates are evaluated in census years by calculating the difference between the value of the estimate and the official census count (Siegel, 2002: 471-479). The difference is error. Expressing the difference as a percent and then calculating the mean percent error for all counties or places yields a summary measure of the bias in the estimates. A negative value means the populations, on average, were underestimated; and a positive value means that the estimates tended to be high. The closer the average is to a value of zero, the less bias in the estimates. This measure of bias is called the Mean Algebraic Percent Error, or MALPE for short. Another way to express bias in estimates is to calculate the percent of positive differences that is, what proportion of the estimates were high. Here a value close to 50% means there is little bias--that is a tendency to over or under estimate.

A second group of summary measures of error are intended to assess the precision of the estimates. If the estimates are in error by substantial differences yet the errors are equally balanced as positive and negative the MALPE and % Positive Differences will show low or no

bias. In order to summarize the precision of the estimates, that is how far they vary from the census count, Mean Absolute Percent Error, referred to in shorthand fashion as MAPE, is used. By calculating the absolute error and determining the mean value across all counties or places, the precision of the estimates may be determined. The closer to zero the lower the variation in estimates from the census count and the better the precision of the estimates. A closely related summary measure of precision is to count the proportion of estimates that have relatively large errors in percentage terms. A commonly used set of thresholds is errors greater than 5 and 10 percent.

In Table 3, summary measures of the errors of estimate are presented for the official DES 2000 estimates of county and place populations, along with a number of other estimates. The top five rows of data in Table 3 contain summary measures of errors for county level estimates, and the remaining rows are for place level estimates. In order to evaluate the quality of the DES estimates it is useful to compare their summary measures of error with other estimates. The official DES, July 1, 2000 estimates were produced using a weighted average of the Housing Unit Method and Ratio-Correlation Method. The measures of error for the two separate methods are presented. Following the Census 2000, the DES has changed their model for county estimates by no longer using the Ratio-Correlation Method, and replacing it with the Composite Method. In row 4, the measures of error for the new model are presented. This model uses a weighted average of the Housing Unit Method and the Composite Method.

In the lower panel, rows 6 through 13, measures of the accuracy of population estimates for places are presented. The official DES estimates for July 1, 2000 are in row 6, followed by the official estimates for 1990. Following them are the summary measures reported by the U.S. Census Bureau (Harper et al, 2002) for their estimates of places in Arizona for April 1, 2000. Rows 9 through 11 report measures of accuracy of the estimates for places by size of place. Comparing these rows demonstrates whether the quality of the estimates vary by size of places. Finally the last two rows are measures of the error in estimates for places due solely to the model used for places. Since the estimates for places are controlled to the county totals, which are in turn controlled to the state totals, some error in the estimates of places may be due to errors in the estimates of the state and counties, and not a function of the model used for places. These rows represent the error that remains after controlling these estimates not to the state and

county estimates, but rather to the Census 2000 counts for the state and county. These errors are due solely to the model used for estimating population of places.

How good are the DES 2000 estimates and much error in precision and bias are acceptable? In some sense that is a relative question. While total accuracy is a target, realistically there will always be some error. A good way to develop a sense of the current level of accuracy and then to set objectives for improving the quality of the estimates is to compare accuracy of estimates over time and across models.

The MAPE for official DES 2000 estimates of county population is 5.0 percent--the average absolute percent error across the states 15 counties. Almost half the counties had percentage errors greater than 5 percent and only one, Yuma, had an error greater than 10 percent. The bias, as expressed in the MALPE and the % Positive Differences, was very low. An obvious goal for improving the estimates is to improve the precision (lower MAPE) while keeping the bias (MALPE) near zero. Line 4 of Table 3 shows how the new model, implemented by DES after 2000, would have performed if it were in place to produce the official county estimates for 2000. The MAPE would have been 4.5 percent, a substantial reduction from the old model's 5.0 percent, and the bias would still have been very low. These data argue that the DES is moving in the right direction with improving their model. Improving the precision for the third of the counties with errors greater than 5 percent should be a major goal now.

Accuracy of the official DES 2000 estimates for places is lower than for counties, and will likely always be lower due to the large number of small places. Relatively small numeric differences are large percentage wise for small places and therefore it is difficult to reduce overall MAPE for places down to the level of MAPE for counties. The good news is that DES 2000 estimates have a lower MAPE than either the DES 1990 estimates or the US Census Bureau estimates for Arizona places. Again, the movement is in the right direction, toward better precision. However the bias in estimates for places is high as expressed by a 5.2 percent MALPE and 66 percent for % Positive Differences.

The MAPE presented in Table 3 is un-weighted, and that results in treating a 10 percent error for a small place as equivalent to a 10 percent error for a large place. In order to see how much the total MAPE is affected by size, places are divided into three size categories based on Census 2000: large places over 100,000 (n = 9); medium places 5,000 to 100,000 (n = 40); and

small places under 5,000 (n = 38). The MAPE is substantially smaller for large places and largest for small places. Bias is less for large places and there is a strong positive bias for medium and small places.

By controlling estimates of places to estimates of counties and the state, the MAPE for places may have been increased due to errors in the estimates for counties and state. In order to see if the amount of error in estimating population for places would have been reduced if the estimates for counties and state had been accurate, the actual Census 2000 count for the state and for the counties were used as controls in two separate calculations. First the place estimates were controlled to the Census 2000 count for the state, and the MAPE for places actually increased slightly (See Row 12). Second the place estimates were controlled to the 2000 counts for the counties, and the MAPE for places improved slightly but was still not better than the MAPE for the actual estimates. In both cases the amount of bias toward overestimation increased. What these data suggest is that to improve the MAPE for places will require some combination of improved models and data for places, because even if DES were able to estimate the state and county populations accurately there would be no improvement in estimates of place population.

IV. Conclusion

An evaluation of a program of population estimates must keep in mind the purpose to which the estimates are being put, and what is a reasonable standard of comparison. In Arizona, the official state population estimates are the basis upon which $1.6 Billion were distributed in 2003 to county and municipal governments. The importance of accuracy--being timely, objective, without bias and with a high degree of precision--is paramount. On the other hand, the estimates must be carried out under a tight production schedule and in a cost effective manner. The objective in developing a population estimate is to derive a value that would be produced if a full enumeration of the population were to be conducted. The population bases for estimates are data from the prior U.S. Census of Population and Housing.

The DES is using the Housing Unit Method supplemented by the Composite Method in order to estimate county population. For sub-county areas--cities and towns--the Housing Unit Method has been used alone. A recent survey of state demographers (FSCPE, 2003) reported that the most common methods for estimating county population were the housing unit and component methods,

followed by the ratio-correlation and composite methods. For sub-county estimates of municipalities the clear favorite was the housing unit method. The DES' choice of methods is in line with the methods employed by most states.

The major weakness in the Housing Unit Method as implemented by the DES is the reliance on occupancy rates and average household size (persons per household) from the prior census. The Census Bureau's new American Community Survey (ACS) will replace the decennial census as the source of these data for counties and municipalities. Once the survey is fully implemented these data will be produced annually. The first data for larger counties and cities (populations greater than 65,000) will appear in 2006. For smaller municipalities, the data will not be ready until 2010. The ACS holds great promise but it will be several years before it can deliver on that promise for smaller areas.

The quality of the DES' official 2000 estimates were good and the changes made since then were improvements when tested against the Census 2000. However, the DES and POPTAC want to do even better. The major objectives should be to keep bias low, while reducing the average error for all size categories, especially the middle and small municipalities. In addition an objective will be improving the precision of the county estimates for the one-third of the counties with errors greater than 5 percent. In order to improve the accuracy of the estimates for counties and places will require a combination of improved models and data. The next step in the improvement process will be to evaluate the data series that serve as inputs to the estimates process.

References

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Table 1: Error of Estimate for Arizona Counties, 2000

			Error of Estimate	
County	**Estimate**	**Census**	**Numeric**	**Percent**
ARIZONA	5,045,275	5,130,632	-85,357	-1.66%
APACHE	67,725	69,423	-1,698	-2.45%
COCHISE	126,300	117,755	8,545	7.26%
COCONINO	124,575	116,320	8,255	7.10%
GILA	51,175	51,335	-160	-0.31%
GRAHAM	36,350	33,489	2,861	8.54%
GREENLEE	9,325	8,547	778	9.10%
LA PAZ	19,350	19,715	-365	-1.85%
MARICOPA	2,991,250	3,072,149	-80,899	-2.63%
MOHAVE	145,425	155,032	-9,607	-6.20%
NAVAJO	95,300	97,470	-2,170	-2.23%
PIMA	866,125	843,746	22,379	2.65%
PINAL	169,475	179,727	-10,252	-5.70%
SANTA CRUZ	40,075	38,381	1,694	4.41%
YAVAPAI	160,075	167,517	-7,442	-4.44%
YUMA	142,750	160,026	-17,276	-10.80%

Table 2: Error of Estimate for Arizona Places, 2000

Place	Estimate	Census	Error of Estimate Numeric	Error of Estimate Percent
APACHE COUNTY				
Eagar town	4,965	4,033	932	23.1%
St. Johns city	3,560	3,269	291	8.9%
Springerville town	2,105	1,972	133	6.7%
COCHISE COUNTY				
Benson city	4,630	4,711	-81	-1.7%
Bisbee city	6,595	6,090	505	8.3%
Douglas city	17,295	14,312	2,983	20.8%
Huachuca City town	2,090	1,751	339	19.4%
Sierra Vista city	40,830	37,775	3,055	8.1%
Tombstone city	1,680	1,504	176	11.7%
Willcox city	3,750	3,733	17	0.5%
COCONINO COUNTY				
Flagstaff city	62,710	52,894	9,816	18.6%
Fredonia town	1,420	1,036	384	37.1%
Page city	9,570	6,809	2,761	40.5%
Williams city	2,905	2,842	63	2.2%
GILA COUNTY				
Globe city	8,145	7,486	659	8.8%
Hayden town	910	892	18	2.0%
Miami town	2,065	1,936	129	6.7%
Payson town	13,665	13,620	45	0.3%
Winkelman town	440	443	-3	-0.7%
GRAHAM COUNTY				
Pima town	2,230	1,989	241	12.1%
Safford city	9,870	9,232	638	6.9%
Thatcher town	4,615	4,022	593	14.7%
GREENLEE COUNTY				
Clifton town	3,030	2,596	434	16.7%
Duncan town	805	812	-7	-0.9%
LA PAZ COUNTY				
Parker town	2,985	3,140	-155	-4.9%
Quartzsite town	2,170	3,354	-1,184	-35.3%

Table 2: Error of Estimate for Arizona Places, 2000

Place	Estimate	Census	Error of Estimate Numeric	Error of Estimate Percent
MARICOPA COUNTY				
Avondale city	35,850	35,883	-33	-0.1%
Buckeye town	8,650	6,537	2,113	32.3%
Carefree town	2,790	2,927	-137	-4.7%
Cave Creek town	3,955	3,728	227	6.1%
Chandler city	176,970	176,581	389	0.2%
El Mirage city	9,910	7,609	2,301	30.2%
Fountain Hills town	19,105	20,235	-1,130	-5.6%
Gila Bend town	1,750	1,980	-230	-11.6%
Gilbert town	108,745	109,697	-952	-0.9%
Glendale city	211,555	218,812	-7,257	-3.3%
Goodyear city	19,695	18,911	784	4.1%
Guadalupe town	5,400	5,228	172	3.3%
Litchfield Park city	3,960	3,810	150	3.9%
Mesa city	388,185	396,375	-8,190	-2.1%
Paradise Valley town	13,395	13,664	-269	-2.0%
Peoria city	108,295	108,364	-69	-0.1%
Phoenix city	1,249,450	1,321,045	-71,595	-5.4%
Queen Creek town	3,955	4,316	-361	-8.4%
Scottsdale city	207,145	202,705	4,440	2.2%
Surprise city	32,815	30,848	1,967	6.4%
Tempe city	162,000	158,625	3,375	2.1%
Tolleson city	4,690	4,974	-284	-5.7%
Wickenburg town	5,175	5,082	93	1.8%
Youngtown town	2,800	3,010	-210	-7.0%
MOHAVE COUNTY				
Bullhead City city	29,910	33,769	-3,859	-11.4%
Colorado City town	4,390	3,334	1,056	31.7%
Kingman city	20,790	20,069	721	3.6%
Lake Havasu City city	42,680	41,938	742	1.8%
NAVAJO COUNTY				
Holbrook city	5,705	4,917	788	16.0%
Pinetop-Lakeside town	3,625	3,582	43	1.2%
Show Low city	8,575	7,695	880	11.4%
Snowflake town	4,850	4,460	390	8.7%

Table 2: Error of Estimate for Arizona Places, 2000

Place	Estimate	Census	Error of Estimate Numeric	Error of Estimate Percent
Taylor town	2,990	3,176	-186	-5.9%
Winslow city	11,395	9,520	1,875	19.7%
PIMA COUNTY				
Marana town	15,185	13,556	1,629	12.0%
Oro Valley town	29,530	29,700	-170	-0.6%
Sahuarita town	3,580	3,242	338	10.4%
South Tucson city	5,675	5,490	185	3.4%
Tucson city	485,790	486,699	-909	-0.2%
PINAL COUNTY				
Apache Junction city	25,880	31,814	-5,934	-18.7%
Casa Grande city	26,490	25,224	1,266	5.0%
Coolidge city	7,520	7,786	-266	-3.4%
Eloy city	10,970	10,375	595	5.7%
Florence town	14,550	17,054	-2,504	-14.7%
Kearny town	2,550	2,249	301	13.4%
Mammoth town	2,070	1,762	308	17.5%
Superior town	3,500	3,254	246	7.6%
SANTA CRUZ COUNTY				
Nogales city	21,810	20,878	932	4.5%
Patagonia town	985	881	104	11.8%
YAVAPAI COUNTY				
Camp Verde town	8,955	9,451	-496	-5.2%
Chino Valley town	7,860	7,835	25	0.3%
Clarkdale town	3,135	3,422	-287	-8.4%
Cottonwood city	9,405	9,179	226	2.5%
Jerome town	580	329	251	76.3%
Prescott city	36,975	33,938	3,037	8.9%
Prescott Valley town	23,285	23,535	-250	-1.1%
Sedona city	10,265	10,192	73	0.7%
YUMA COUNTY				
San Luis city	16,465	15,322	1,143	7.5%
Somerton city	7,240	7,266	-26	-0.4%
Wellton town	1,425	1,829	-404	-22.1%
Yuma city	73,260	77,515	-4,255	-5.5%

Table 3: Measures of the Accuracy of Population Estimates for Arizona

ROW	COUNTY	MAPE	MALPE	% Positive Differences	Errors >5%	Errors >10%
1	DES, July 1, 2000*	5.0%	0.2%	40%	47%	7%
2	Housing Unit Method	4.9%	-1.2%	40%	40%	7%
3	Ratio-Correlation Method	5.5%	0.1%	53%	40%	20%
4	DES, July 1, 2000+	4.5%	-0.4%	47%	33%	0%
5	Composite Method	5.9%	-1.6%	40%	53%	13%
	PLACES					
6	DES, July 1, 2000	9.7%	5.2%	66%		
7	DES, July 1, 1990	17.1%	-5.9%			
8	Census Bureau Estimates, April 1, 2000	11.6%	0.3%			
9	DES 2000 - Large Places	1.8%	-0.8%	33%	11%	0%
10	DES 2000 - Medium Places	8.7%	5.2%	70%	58%	28%
11	DES 2000 - Small Places	12.7%	6.6%	66%	74%	45%
	Controlled to Census 2000					
12	State Total	10.6%	7.0%			
13	County Totals	10.0%	6.4%			

* Old Model using Weighted Average of Housing Unit Method and Ratio-Correlation Method

+ New Model using Weighted Average of Housing Unit Method and Composite Method

MAPE - Mean Absolute Percent Error, a measure of accuracy, smaller is better

MALPE - Mean Algebraic Percent Error, a measure of bias, zero is goal, more positive means higher overestimate on average, and more negative means higher underestimate on average

% Positive Differences - Another measure of bias, 50% is goal, higher means more overestimates than underestimates

Errors >5% - A measure of tendency to large errors

Errors >10% - A measure of tendency to extreme errors